DEHN

AD A090382

LEVEL

THE PARTICLE DYNAMICS OF PENETRATION

JAMES DEHN, DR.
ARRADCOM, BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MD. 21005

Mathematical modeling of the target penetration process is an old field and the great variety of models we now have reflects this fact. At one end of the model spectrum we have simple empirical interpolation formulas which can serve as convenient summaries of what we know already, though they cannot further our understanding. At the other extreme we have very complicated continuum models which include all the science we think is appropriate and offer the possibility of prediction without further experimentation. However, such advanced models have a number of shortcomings at present. On the one hand, many of the material properties needed to implement them have not been measured, while on the other, the numerical methods used in solving the equations involved are not yet sufficiently advanced to provide either rapid or routine calculational tools. Consequently, they have not yet improved our understanding of penetration very much, at least in the ordnance range. Between these two extremes there are a number of intermediate approaches which are based on simplified physical laws and so offer the possibility of improved understanding together with a calculational tool which can be routine, rapid and reliable.

The oldest type of simplified physical model in use consists of replacing the projectile by a mass point and the target by a force field. If the striking mass, m_o, is constant in time, then the equation

$$m_o\dot{v} = F = -(a + b_1v + b_2v^2) \qquad (1)$$

can be used to describe certain types of rectilinear motion. Here a,

b_1 and b_2 are constants so the force, F, is a quadratic form in the velocity, v. Euler and Robins in 1742 applied this equation to target penetration with $b_1 = b_2 = 0$. In 1830 Poncelet did the same with only $b_1 = 0$, while Résal continued this tradition in 1895 by applying the solution with only a = 0. Today we know that both the velocity, $v = \dot{s}$, and the displacement, s, can be found explicitly as closed form functions of time by standard integrations with none of the constants zero.

In applications of equation (1) to penetration problems, the force is taken antiparallel to the velocity and the "normal" case of zero obliquity impact is discussed. Thus equation (1) is limited to one-dimensional motion, since there is no force component to make the mass deviate from a straight line. This is a severe limitation since curvilinear motion, including ricochet, is known to occur. Even for rectilinear motion the force in equation (1) cannot describe embedment of the projectile in the target unless a = 0. If $a \neq 0$, then F can never vanish and the mass can come to rest only momentarily instead of permanently as occurs in the case of embedment.

The constant a in equation (1) has been interpreted as the force needed to detach a certain amount of target material and move it out of the way of the projectile as a single plug or in some other manner. Since a projectile of larger cross-sectional area, A, must move more target material, this force is usually taken to be proportional to A. Similarly, since a thicker target plate will require the removal of more material for a given A, we might also take this force proportional to the target thickness, T. Thus, the product, AT, becomes a lower bound on the volume of target material which is removed during a complete perforation. If the projectile striking speed, v_o, is increased, then in general more momentum will be transferred to the material which is removed, so it is also reasonable to assume a dependence of this force on v_o as well. In this paper we will use the simple assumption that

$$a = a_o T + a_1 v_o \qquad (2)$$

where a_o and a_1 depend on the properties of the target and projectile as well as the projectile shape.

Another interpretation for part of this force is the frictional resistance offered by the target which depends on the area of contact rather than the cross-sectional area. It also depends on the pressure in the simplest approximation and so also depends on v_o.

The term b_1v in equation(1) can be called a viscous force. This does not imply the presence of the liquid state. The terminology, "viscosity of solids" is well established and only implies a proportionality between shear stress and time rate of strain. The viscosities of various solids have been measured up to explosive rates of motion and a discussion of these experiments has been given by Walters[1]. This force also will depend on the geometry of the projectile. For a sphere moving through a viscous liquid, for example, Stokes' law gives us a force which depends on the radius of the sphere as well as the speed.

The term b_2v^2 in equation (1) can be called a drag force. Again this does not imply the presence of a liquid. Drag forces in solids are not well studied, but they should also depend on the cross-sectional area of the projectile. In an isotropic fluid such as still air, the components of the drag force are $b_2v\dot{x}$ and $b_2v\dot{z}$ for the two coordinates x and z with $v^2 = \dot{x}^2 + \dot{z}^2$. In an anisotropic medium we might use $b_{2x} \neq b_{2z}$ or the components of the speed instead of the speed, or both. Since penetration problems of interest are very anisotropic because of the presence of air-metal interfaces which make the resistance parallel or perpendicular to such interfaces quite different, we propose the form $b_{2s}\dot{s}^2$ with s = x,y,z in general.

Several of the forces described above involve areas which can depend on the depth of penetration. For example, for a projectile with a curved nose the cross-sectional area will increase as it penetrates more deeply, while for a long rod the contact area will increase with penetration depth. Such behavior will not generally be simply proportional to s. If the trajectory is curved the situation may change in time and when a projectile enters its final breakout phase before exiting the target in a ricochet or perforation, the various forces being described may decrease as the penetration progresses. In any case the amount of target material moved out of the way in time t depends on s, the current length of the trajectory. For simplicity we will assume a form cs with s = x,y,z and c constant.

This line of reasoning suggests that we consider component equations of motion of the form

$$m_o\ddot{s} + F_s = m_o\ddot{s} + a_s + b_{1s}\dot{s} + b_{2s}\dot{s}^2 + c_s s = 0 \qquad (3)$$

where s = x,y,z in a rectangular coordinate system with origin at the point of impact. For a plate target let us take the z axis pointing along the line of flight of a projectile which impacts at zero obliquity and choose the x axis so that an oblique striking velocity vector lies in the x,z plane. Then the y coordinate is ignorable for such a target which is "effectively infinite" in the x,y plane. Analysis shows[2] that we can neglect the $b_{2s}\dot{s}^2$ term in equation (3) and still describe ricochet, embedment and perforation. However, we cannot do this if we neglect the $c_s s$ term. This is fortunate since we can find an exact solution for equation (3) if we neglect $b_{2s}\dot{s}^2$. Such a solution should be appropriate for lower speeds like those of ordnance interest and can be amended if we wish to include hypervelocity impacts. Without the $b_{2s}\dot{s}^2$ term the solution is

$$s = A_{1s}^{+} e^{\gamma_s^{+} t} + A_{1s}^{-} e^{\gamma_s^{-} t} + \Delta_s \tag{4}$$

where $A_{1s}^{\pm} = (v_{os} + \Delta_s \gamma_s^{\mp})/(\gamma_s^{+} - \gamma_s^{-})$ for initial conditions $s = 0$ and $\dot{s} = v_{os}$ at $t = 0$. Here $v_{ox} = v_o \sin\theta_o$ and $v_{oz} = v_o \cos\theta_o$ where θ_o is the striking obliquity measured counterclockwise from the negative z axis to v_o. In addition $\gamma_s^{\pm} = -\alpha_s \pm \sqrt{\alpha_s^2 - c_s/m_o}$ where $\alpha_s = b_{1s}/(2\,m_o)$, and $\gamma_s^{-} < \gamma_s^{+} < 0$ if s and $\dot{s}$ always remain finite. The degenerate case of $\alpha_s^2 = c_s/m_o$ has a special solution[2]. Positive c_s and real $\gamma_s^{\pm}$ requires $0 < c_s < m_o\alpha_s^2$.

The constants Δ_s in equation (4) are position components of a stable node and have the form

$$\Delta_s = -a_s/c_s = -(a_{os}T_s + a_{1s}v_{os})/c_s \tag{5}$$

when we use equation (2) in component form. Here $T_z = T$ while $T_x = T\tan\theta_o$. If we include the $b_{2s}\dot{s}^2$ term we can write approximate solutions by using standard perturbation theory or we can construct closed forms which reduce to the known solutions when $b_{2s} \to 0$ or $c_s \to 0$. We will not discuss such matters here. Instead, let us consider the force components at the ricochet limit velocity, v_{oRL}, and at the perforation limit velocity, v_{oPL}. At the ricochet limit $v_{oz} = v_{ozRL}$ and $F_z = z = \dot{z} \to 0$ as $t \to \infty$, so (even if we retain the drag force)

$$F_z \to 0 = -[a_{oz}T + a_{1z}v_{ozRL}] \tag{6}$$

and at the perforation limit, $v_{oz} = v_{ozPL}$, $z \to T$ and $F_z = \dot{z} \to 0$ as $t \to \infty$, so

$$F_z \to 0 = -[a_{oz}T + a_{1z}v_{ozPL} + c_z T]. \tag{7}$$

From equations (6) and (7) we can find a_{oz}/c_z and a_{1z}/c_z in terms of v_{ozRL}, v_{ozPL} and T. If we use these values in equation (5), we obtain

$$\Delta_z = T(v_{oz} - v_{ozRL})/(v_{ozPL} - v_{ozRL}). \tag{8}$$

If $v_{oz} < v_{ozRL}$, then $\Delta_z < 0$ and ricochet will occur.

If $v_{oz} > v_{ozPL}$, then $\Delta_z > T$ and perforation will occur.

If $0 < \Delta_z < T$, then embedment will occur. Thus Δ_z, the z-component of the final position, is a convenient index for predicting the eventual outcome. A similar procedure for the x-component of the force gives

$$\Delta_x = [(x_{AL} v_{oxPL} - x_{PL} v_{oxRL}) + (x_{PL} - x_{RL}) v_{ox}] / (v_{oxPL} - v_{oxRL}). \quad (9)$$

For the z-component we had the ricochet limit position $z_{RL} = 0$ and the perforation limit position $z_{PL} = T$. Similarly, x_{RL} and x_{PL} are the x-components of these limit positions and Δ_x is the x-component of the final position towards which the motion tends. Since all of these limit positions and velocities are measurable quantities, then a_{os}/c_s, a_{1s}/c_s and so a_s/c_s (or Δ_s) for s = x,y can be determined experimentally.

As mentioned above, the b_{1s} parameters can be estimated from viscosity experiments. For a sphere we can take advantage of the fact that Stokes' law is proportional to the speed and write $b_{1s}\dot{s} \sim R\mu\dot{s}$ where R is the radius and μ is the dynamic (density-dependent) viscosity as discussed by Walters[1]. For other shapes we might expect somewhat different forms. If we neglect the b_{2s} parameters we are ready to calculate trajectories, exit speeds and exit angles for a variety of conditions.

Figures 1a and 1b show the exit speed and exit angle of a one gram steel sphere which strikes a 9.53 mm thick aluminum plate at $\theta_o = 45°$ over a range of v_o which covers the ricochet, embedment and perforation regions. The data points were obtained by Backman and Finnegan[3], while the solid lines were calculated by using the time derivatives of the components in equation (4) with the parameters indicated in the Figures. The exit speed is $v = \sqrt{\dot{x}^2 + \dot{z}^2}$ while the exit angle, $\theta = \arctan(\dot{x}/\dot{z})$, is measured from the z axis in the counterclockwise direction. For perforation, $\theta < 90°$, while for ricochet $\theta > 90°$. Similar calculations for other striking obliquities from 0 to 60 degrees using the same parameters also show general agreement with experiment. Comparisons for other materials and other target and projectile geometries are in progress. If we retain the $b_{2s}\dot{s}^2$ term in equation (3), the agreement with experiment can be improved.

Now let us extend our linear model to an eroding penetrator. For a constant mass penetrator energy is dissipated through the

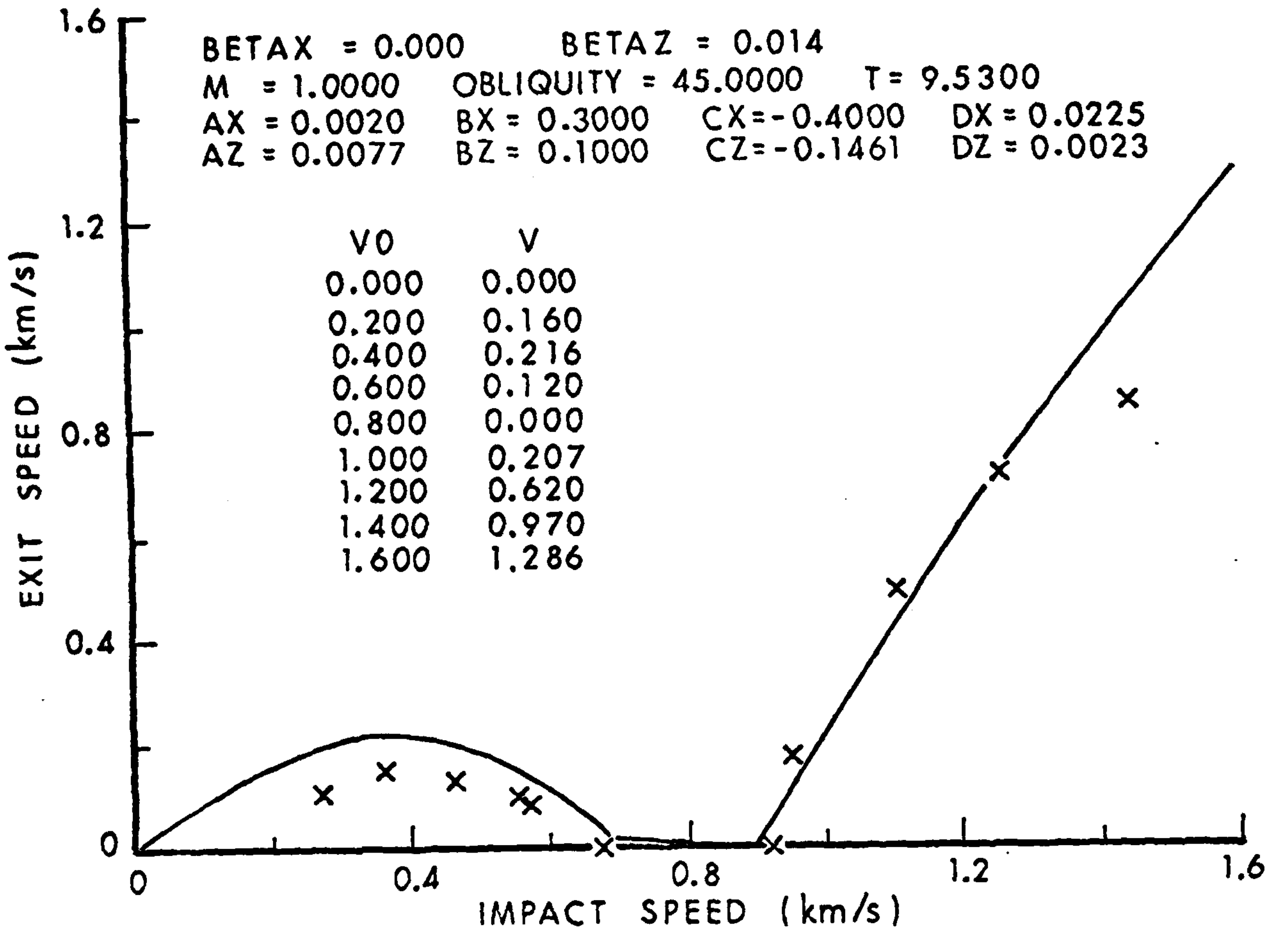

Figure 1a. Measured and calculated exit speeds for steel spheres vs aluminum plates at 45° obliquity.

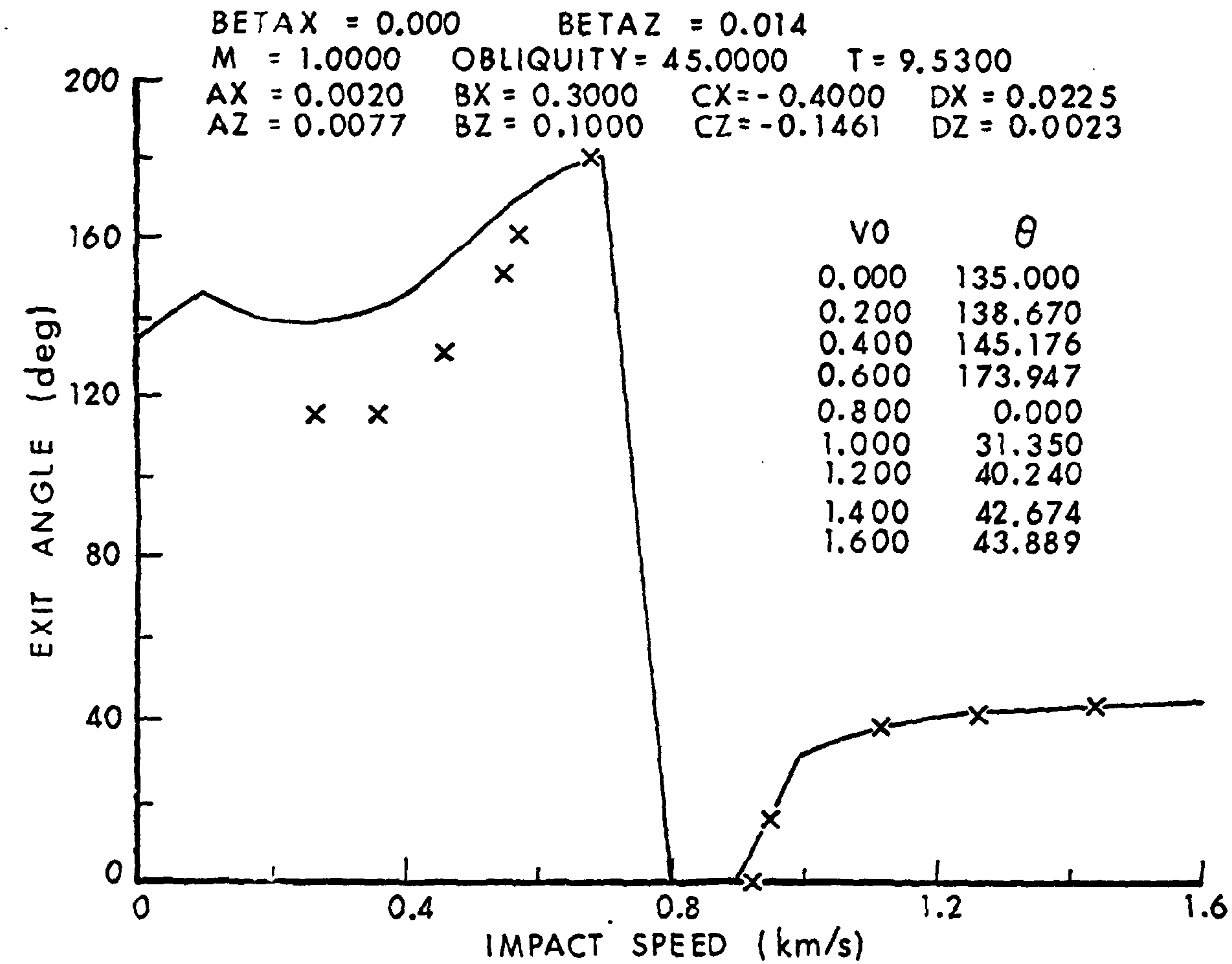

Figure 1b. Measured and calculated exit angles for steel spheres vs aluminum plates at 45° obliquity.

viscous term and shows up as a decrease in velocity. If $\dot{m} \neq 0$, energy can also be dissipated by a decrease in mass. Since the only dissipative term we have in our force field is the viscous term, we might consider amending this term to reflect this additional means of dissipating energy. If we modify this term to be $(b_{1s} - \varepsilon_s \dot{m})\dot{s}$ where ε_s is a positive, dimensionless constant, we have a form which reduces to our previous form when $\dot{m} = 0$. If $\dot{m} < 0$ and mass is lost, then this term is larger, and if $\dot{m} > 0$ it is smaller. Cases of $\dot{m} > 0$ could include precipitation of various kinds, while cases of $\dot{m} < 0$ could include re-entry shield ablation, penetrator erosion and a variety of other phenomena. Our equation, neglecting drag, becomes

$$m\ddot{s} + \dot{m}\dot{s} + a_s + (b_{1s} - \varepsilon_s \dot{m})\,\dot{s} + c_s\, s = 0 \tag{10}$$

which reduces to equation (3) for $\dot{m} = b_{2s} = 0$. If we make an arbitrary transformation of the independent variable, time, to a new independent variable, ϕ, we find

$$Ms'' + B_s s' + a_s + c_s s = 0 \tag{11}$$

where a prime means $d/d\phi$ and

$$M = m\dot{\phi}^2 \tag{12}$$

while

$$B_s = [m\dot{\phi}' + b_{1s} + (1-\varepsilon_s)\dot{m}]\,\dot{\phi}. \tag{13}$$

Since we are free to choose $\phi(t)$, let us choose it so M and B_s in equations (12) and (13) are constants. Then $\dot{m} = -2m\dot{\phi}'$ from equation (12) so we can eliminate $m\dot{\phi}'$ from equation (13) and obtain $B_s = [b_{1s} + (.5-\varepsilon_s)\dot{m}]\dot{\phi} = [b_{1s} + (.5-\varepsilon_s)\dot{m}_o]\,\dot{\phi}_o$ since B_s is constant. In order that $B_{1s} \to b_{1s}$ in the constant mass case, $\dot{m} = \dot{m}_o = 0$, we must have $B_{1s} \to b_{1s}\dot{\phi}_o = b_{1s}$ or $\dot{\phi}_o = 1$. Thus $M = m\dot{\phi}^2 = m_o\dot{\phi}_o^2 = m_o$. For this reason we will call this choice of ϕ a constant mass transformation. If we eliminate m and $\dot{m}$ from equation (13) in favor of $\dot{\phi}'$ and $\dot{\phi}^2$ we can integrate with respect to ϕ to obtain $\dot{\phi}$ and then with respect to t to obtain

$$(b_{1s}/B_s)\,\phi + D_s(1-b_{1s}/B_s)(1 - e^{-\phi/D_s}) = t \tag{14}$$

as the transformation which makes M and B_s constant. Here $D_s = m_o(2\varepsilon_s - 1)/B_s$. If we use $\dot{\phi}$ in equation (12), we find

$$m = m_o\,[(1-b_{1s}/B_s)e^{-\phi/D_s} + b_{1s}/B_s]^2 \tag{15}$$

and the solution of equation (11) is

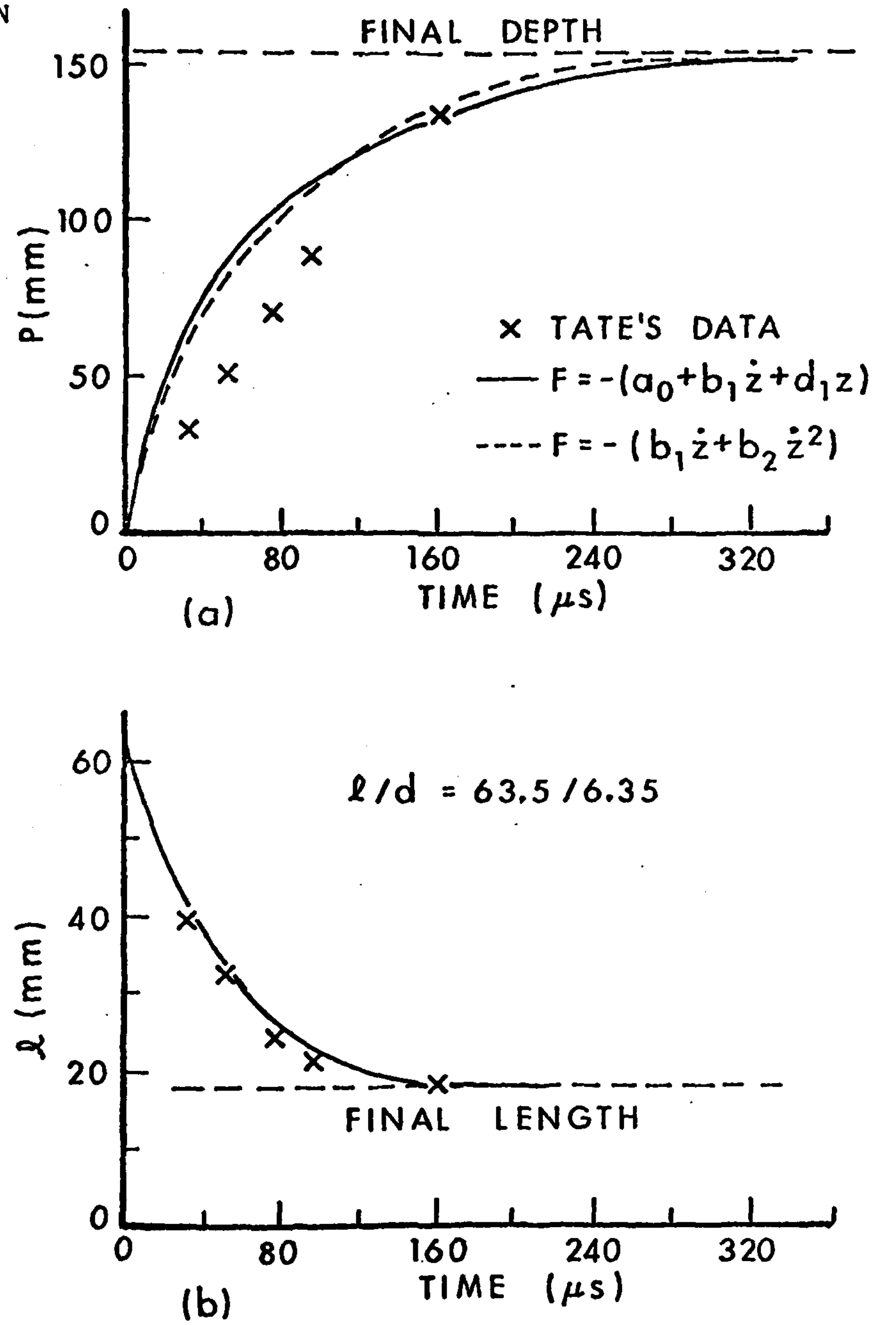

Figure 2 Dural rod penetrating polyethylene: measured and calculated values.

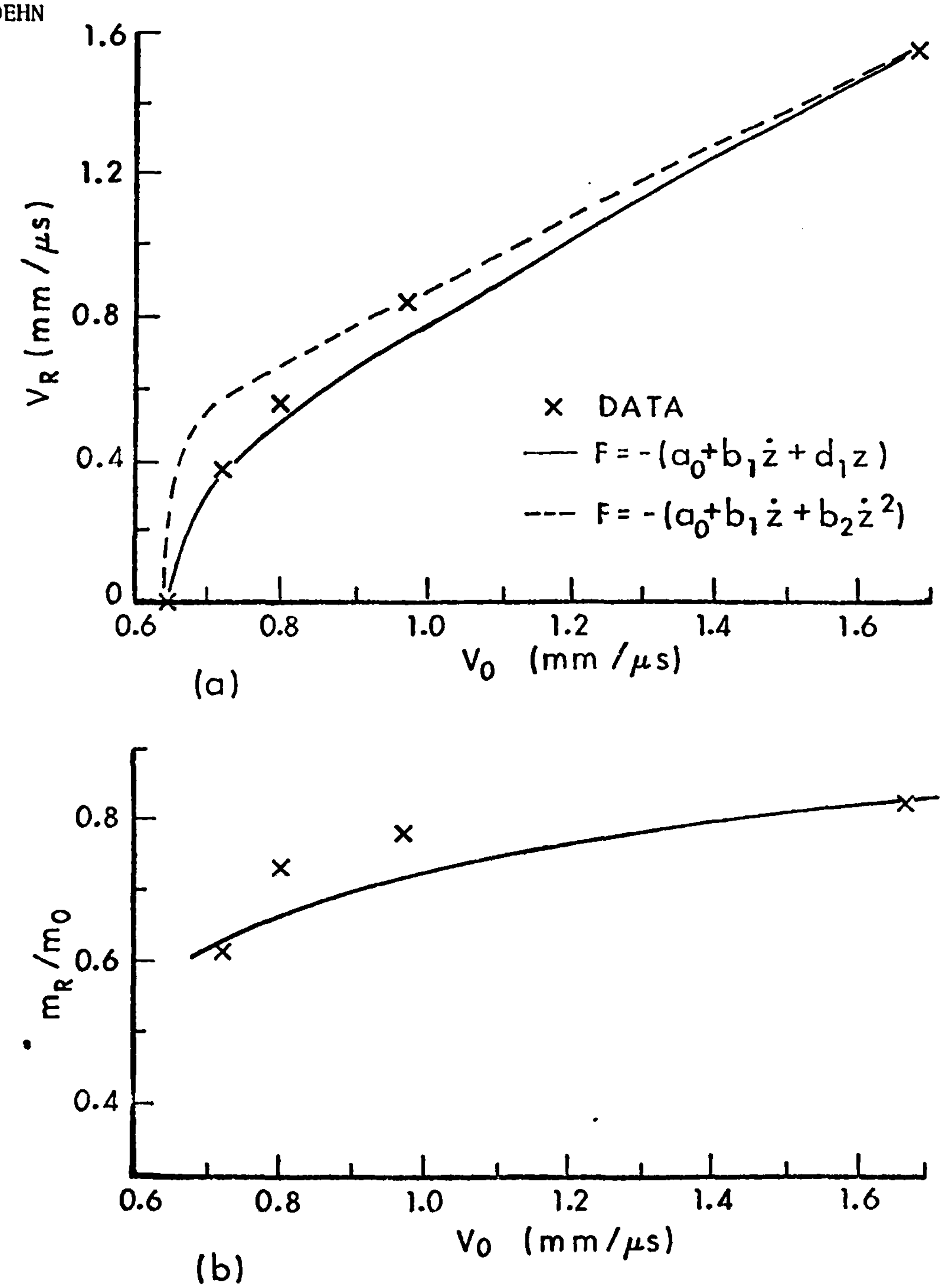

Figure 3. Residual speed and mass versus striking speed for a long steel rod penetrating a steel plate: measured and calculated values.

$$s = A^{+}_{2s}\, e^{\lambda^{+}_{s}\phi} + A^{-}_{2s}\, e^{\lambda^{-}_{s}\phi} + \Delta_s \qquad (16)$$

with the same mode Δ as equation (4). Here $A^{\pm}_{1s}$ and $\lambda^{\pm}_{s}$ are analogous to $A^{\pm}_{1s}$ and $\gamma^{\pm}_{s}$ above.

If we analyze the case $\dot{m} < 0$, we find[2] that for $\varepsilon_s > .5$ and $b_{1s} > 0$, m will not vanish, although it may become very small if $b_{1s}/B_s << 1$. For $\varepsilon_s < .5$ m will always vanish in a semi-infinite target. Since penetrators do not always vanish in a semi-infinite target, we see why $\varepsilon_s = 0$ in equation (10) (a particular case of $\varepsilon_s < .5$) is not a general enough case to apply to penetration.

We have applied the particular solution represented by equations (14), (15), and (16) to a variety of problems. A special case of a frictionless harmonic oscillator with variable mass is a liquid-filled vessel of constant horizontal cross-section with a hole in the bottom through which it loses its contents as it oscillates on a spring in the earth's gravitational field. Here we will merely mention the result[2] that the motion and its variable periodicity can be described over many cycles from full to nearly empty vessel with an accuracy of better than 5%. Our main interest here is to apply this solution to an eroding penetrator.

Figures 2a and 2b give the depth of penetration, $P = z+\ell/2$, of the tip of a dural rod of initial length $\ell_o = 63.5$ mm and constant diameter $d = 6.35$ mm and the variable length $\ell = m/[\pi\rho d^2/4]$ versus time in a semi-infinite polyethylene target. The data was obtained by Tate[4] while the calculations employed equations (14),(15) and (16). An alternate force calculation compatible with equations (14) and (15) gave the dashed curve.

Figures 3a and 3b give the exit speed and the fractional mass remaining after perforation of a 6.35 mm thick steel plate by a 7.78 gm steel rod (ℓ_o/d = 50 mm/5 mm) striking end on at zero obliquity and various speeds, v_o. The data were obtained by Herr and Grabarek[5] while equations (14), (15) and (16) were used for the calculation. An alternate force calculation compatible with equations (14) and (15) gave the dashed curve. For simplicity the initial erosion rate was taken to be independent of v_o, although better agreement could have been obtained by assuming that $\dot{m}_o$ is a function of v_o. Since $\dot{m}$ declines only slightly during the brief times required for perforation over the measured range of v_o, we have a simple explanation for the observed fact that the length of rod remaining increases as v_o increases (for a fixed target thickness). At higher v_o the projectile spends less time in the target so that less erosion takes place (since the rate is approximately constant). Other types of behavior are also allowed by the model[2].

In this paper we have discussed a model which describes the particle dynamics of penetration in a manner which can improve our understanding and provide us with a routine, rapid and reasonably reliable method of calculation. Once a library of experimental results has been established we should be able to interpolate and extrapolate with considerable confidence. The model describes the main features of oblique penetration, namely exit speed, angle and mass, over the entire region of interest, including richochet embedment and perforation. Applications to multiple plate targets and other geometries should also be possible. In addition, other features of interest such as projectile breakup[2] can be linked with this model.

REFERENCES

1. W.P. Walters, "Influence of Material Viscosity on the Theory of Shaped-Charge Jet Formation," ARBRL-MR-02941, August 1979.

2. J.Dehn, "The Particle Dynamics of Target Penetration," ARBRL-TR-02188, September 1979.

3. M.E.Backman and S.A.Finnegan, "Dynamics of the Oblique Impact and Ricochet of Nondeforming Spheres Against Thin Plates," NWC TP 5844, September, 1976.

4. A. Tate, "A Theory for the Deceleration of Long Rods after Impact", J. Mech. Phys.Solids 15, 387 (1967).

5. L. Herr and C.Grabarek," Ballistic Performance and Beyond Armor Data for Rods Impacting Steel Armor Plates," BRL MR 2575, January 1976.